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THE TURBULENT BOUNDARY LAYER ON A ROUGH

CURVILINEAR SURFACE

By V. F. Droblenkov

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A number of semiempirical approximate methods exist for determining the characteristics of the turbulent boundary layer on a curvilinear surface. At present, among these methods, the one proposed by L. G. Loitsianskii (ref. 1) is given frequent practical application. This method is sufficiently effective and permits, in the case of wing profiles with technically smooth surfaces, calculating the basic characteristics of the boundary layer and the values of the overall drags with an accuracy which suffices for practical purposes.

The idea of making use of the basic integral momentum equation

$$\frac{d\delta^{xx}}{dx} + \frac{V'\delta^{xx}}{V}(2 + H) = \frac{\tau_0}{\rho V^2} \qquad (1)$$

proves to be fruitful also for the solution of the problems in the determination of the characteristics of the turbulent boundary layer on a rough surface. In fact, using, for instance, the well-known values of the averaged longitudinal velocities of the turbulent layer on a plate with a developed roughness

$$\frac{v^0}{v_*} = 2.5 \ln \frac{y}{k_s} + 8.48$$

the equation for the relationship between the local-friction coefficient c_f and δ^{xx}/k_s may be obtained in the form

$$\frac{1}{\sqrt{\tau_0/\rho V_0^2}} = 5.75 \lg\left(\frac{\delta^{xx}}{k_s}\right) - 5.75 \lg\left[\left(\frac{\tau_0}{\rho V_0^2}\right)^{1/2} - 5\frac{\tau_0}{\rho V_0^2}\right] + 6.20 \qquad (2)$$

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*"Turbulentnyĭ pogranichnyĭ sloĭ na sherpokhovatoĭ krivolineĭnoĭ poverkhnosti." Izvestiia Akademii Nauk SSSR, Otdelenie tekhnicheskikh nauk [Reports of the Academy of Sciences of the S.S.S.R., Department of Technical Sciences], no. 8, 1955, pp. 17-21.

Here and above, the following symbols are used:

$$c_f = \frac{\tau_0}{\frac{1}{2}\rho V_0^2}$$

$$\delta^{xx} = \int_0^{\delta,\infty} \frac{v^0}{V}\left(1 - \frac{v^0}{V}\right)dy$$

$$\delta^x = \int_0^{\delta,\infty} \left(1 - \frac{v^0}{V}\right)dy$$

$$H = \frac{\delta^x}{V}$$

$$v_* = \left(\frac{\tau_0}{\rho V_0^2}\right)^{1/2}$$

where δ^{xx} is the momentum thickness loss, δ^x the displacement thick-
ness, k_s the height of the protuberances of the equivalent roughness,
V_0 the velocity of the approach flow, V the velocity on the upper
limit of the boundary layer, and v^0 the velocity of the inside layer.

Figure 1 represents $L_1 = \lg\left(\rho V_0^2/\tau_0\right)$ as a function of $\lambda_1 = \lg R^{xx}$
and $\lambda_2 = \lg\left(\delta^{xx}/k_s\right)$ where curve 1 pertains to the laminar layer and
curve 2 to the turbulent boundary layer for which

$$\frac{\tau_0}{\rho V_0^2} = 0.00655\left(R^{xx}\right)^{-1/6}$$

Both curves refer to the case of a technically smooth surface, whereas
curve 3 applies to the turbulent boundary layer

$$\frac{\tau_0}{\rho V_0{}^2} = 0.0031\left(\frac{\delta^{xx}}{k_s}\right)^{-1/6} \tag{3}$$

in the case of a rough surface.

As can be seen from figure 1, the equality (3), replacing the equality (2), is correct over the wide range of variation of the relative roughness[1] and is more convenient for practical calculations. With the aid of the equality (3), the solution of the fundamental boundary-layer equation (1) no longer presents any difficulty for rough plane surfaces or for rough curvilinear surfaces.

Using the equality (3), the fundamental momentum equation (1) for the case of a longitudinal flow about the plates $\left(V = V_0, \quad V' = 0\right)$

$$\frac{d\delta^{xx}}{dx} = \frac{\tau_0}{\rho V_0{}^2}$$

is rewritten in the form

$$\frac{d\delta^{xx}}{dx} = 0.0031\left(\frac{\delta^{xx}}{k_s}\right)^{-1/6}$$

By integration the following equation is obtained:

$$\left(\delta^{xx}\right)^{7/6} = \frac{7}{6}\, 0.0031\left(k_s\right)^{1/6} x + C$$

Taking into consideration that the turbulent boundary layer is established from the leading edge $\left(x_0 = 0, \quad \delta_0{}^{xx} = 0\right)$, $C = 0$ is obtained, and finally

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[1]If necessary, the equality (2) can be approximated by an expression which is, as to form, analogous to the equality (3), but reflects much more strongly the dependence of (2) in the case of essentially different values of k_s and c_f.

$$\delta^{xx} = 0.0080\left(k_s\right)^{1/7}x^{6/7} \quad \text{or} \quad \frac{\delta^{xx}}{x} = 0.0080\left(\frac{x}{k_s}\right)^{-1/7}$$

According to the value δ^{xx}, the local turbulent-friction coefficient on an entirely rough plate can be found as

$$c_f = \frac{\tau_0}{\frac{1}{2}\rho V_0^2} = 0.0062\left(\frac{\delta^{xx}}{k_s}\right)^{-1/7} = 0.0139\left(\frac{x}{k_s}\right)^{-1/7}$$

and the value of the overall drag coefficient of a rough plate of a given length L as

$$C_f = \frac{R}{\frac{1}{2}\rho V_0^2 L} = \frac{1}{\frac{1}{2}\rho V_0^2 L}\int_0^L \tau_0 \, dx = \int_0^L c_f \, d\left(\frac{x}{L}\right) = 0.0162\left(\frac{L}{k_s}\right)^{-1/7} \quad (4)$$

The final results of the calculations, carried out according to the equality (4), are plotted in figure 2 where the solid line represents C_f as a function of $\lg\left(L/k_s\right)$, obtained by calculation according to formula (4), and the dotted line represents Schlichting's data. As can be seen, the calculated results show good agreement with the proved values of the overall turbulent-friction coefficients of plates for the regime of developed roughness obtained by Schlichting. This latter fact permits us to consider the obtained solutions as correct over a wide range of variation of relative roughness.

With the aid of equality (3) and the momentum equation (1) the turbulent boundary layer can also be calculated on a rough surface in the case of a longitudinal pressure drop.

Let it be assumed that for the case of a turbulent boundary layer on a rough surface, there exists the function $G_{sh}\left(k_s; \delta^{xx}\right)^*$ which has

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*NACA reviewer's note: The subscript sh stands for the Russian word "sherpokhovatoi," which translated, means "rough."

the property that, when multiplied by the quantities $V'\delta^{xx}/V$ and $\tau_0/\rho V^2$, it becomes independent of the Reynolds number and the parameter

$$f = \frac{V'\delta^{xx}}{V} G_{sh} \tag{5}$$

and its function

$$\zeta_{sh}(f) = \frac{\tau_0}{\rho V^2} G_{sh} = \zeta_{sh} \tag{6}$$

is also independent of the quantity k_s/δ^{xx}.

Determination of the form of the function G_{sh} is carried out here also, as in L. G. Loĭtsianskiĭ's method, by starting from the analogy with the laminar boundary layer, under the assumption that for all values f the form of the function G_{sh} coincides with that for a turbulent flow, in the case of a longitudinal flow about a rough plate.

Since V' is equal to 0 on the plate, according to the equalities (5) and (6) on the plate, $f = 0$ and $\zeta_{sh} = $ const. Then, assuming for simplification $\zeta_{sh} = $ const. $= 1$, according to the equalities (6) and (3)

$$1 = \frac{\tau_0}{\rho V_0^2} G_{sh} = 0.0031\left(\frac{\delta^{xx}}{k_s}\right)^{-1/6} G_{sh}$$

Hence, there is found, also, the form of the function G_{sh}:

$$G_{sh} = A\left(\frac{\delta^{xx}}{k_s}\right)^m = 323\left(\frac{\delta^{xx}}{k_s}\right)^{1/6} \tag{7}$$

The value of the argument of the function G_{sh} for every section of the boundary layer of the surface under consideration is determined from the solution of the fundamental equation (1). This solution may be represented by an expression analogous to the solution obtained by L. G. Loĭtsianskiĭ for a turbulent regime of flow about a technically smooth surface. Following his method, after several transformations, instead of the equality (1), an ordinary differential equation can be obtained for determining f on the rough surface

$$\frac{df}{dx} = \frac{V'}{V} F(f) + f \frac{V''}{V} \tag{8}$$

where

$$F(f) = (1 + m)\zeta_{sh} - f\left[3 + 2m + (1 + m)H\right] = \frac{7}{6} \zeta_{sh} - f\left[\frac{20}{6} + \frac{7}{6} H\right]$$

$$m = \frac{1}{G_{sh}}\left(\frac{\delta^{xx}}{k_s}\right)G_{sh}' = \frac{1}{6}$$

Since the solution of equation (8) is known, it is not difficult to find δ^x, δ^{xx}, and $\tau_0/\rho V_0^2$ if the form of the function G_{sh} for every section of the turbulent boundary layer on a rough surface is known, as well as that for the function $F(f)$.

If the longitudinal drop is small, the values of f also will be small, and it will not involve a great inaccuracy to substitute for H and ζ_{sh}, in the equality for $F(f)$, their values for $f = 0$.

Then, assuming that $H(f)$ is equal to 1.3 for $f = 0$ $\left(\text{for large Reynolds numbers and } x/k_s > 1 \times 10^4\right)$ and ζ_{sh} is equal to 1.0, it follows that

$$F(f) \approx a - bf = 1.17 - 4.85f$$

and the solution of the fundamental equation of the turbulent boundary layer on a rough surface is used for determining the quantity $f(x)$ in the form of a simple quadrature

$$f(x) = \frac{V'}{V^b}\left[a \int_0^x V^{b-1}dx + c\right]$$

or

$$f(x) = a\frac{V'}{V^b}\int_0^x V^{b-1}dx = \frac{1.17}{V^{4.85}}\int_0^x V^{3.85}dx$$

where the turbulent boundary layer begins directly at the leading edge.

The calculation of the values of $f(x)$ according to the values of the velocities V on the upper limit of the boundary layer does not present any difficulties.

With the aid of the equalities (5) and (7) the values of the momentum thickness loss at the trailing edge are found to be

$$\delta_k^{**} = \left[\frac{ak_s^m}{AV_k^{b-1}}\int_0^{x_k} V^{b-1}dx\right]^{\frac{1}{m+1}}$$

Consequently, the value of the total-drag coefficient of a body with a rough surface in a two-dimensional, completely steady, turbulent flow regime becomes

$$C_x = 2\frac{\delta_k^{**}}{L}\left(\frac{V_k}{V_0}\right)^{2+\frac{1}{2}(H_k+1)} = \frac{2}{L}\left[\frac{ak_s^m}{AV_k^{b-1}}\int_0^{x_k} V^{b-1}dx\right]^{\frac{1}{m+1}}\left(\frac{V_k}{V_0}\right)^{2+\frac{1}{2}(H_k+1)} \tag{9}$$

For the purpose of additional verification and proof of the correctness of the resulting conclusions regarding the equalities (3), (4), and (9), the values C_x of the NACA 0012 wing profile were calculated for three surface conditions: technically smooth, rough with the value $L/k_s = 8.12 \times 10^3$, and rough with $L/k_s = 2.03 \times 10^4$. The results of the calculation were compared with the results of tests in a variable-density tunnel with a wing of that profile, an aspect ratio of 6, and a chord length of 203 mm. The test data and calculated results are presented in figure 3. There, the curve I corresponds to the drag coefficient of a technically smooth plate in the case of a completely turbulent boundary layer; to the measured values of C_x for the wing profile, curve II corresponds to an accurately handmade surface structure; and curves III and IV represent conditions for surfaces covered with grains

of carborundum powder of the size $k_s = 0.025$ mm and $k_s = 0.01$ mm, respectively. As can be seen from the data of figure 3, the calculated results (solid lines 1, 2, and 3) and the test results (dotted lines) agree with each other sufficiently well for high Reynolds numbers when a turbulent regime of flow is established along practically the entire profile length. The insignificantly excessive values of the calculated quantities C_x (obtained for a completely turbulent flow regime) compared to the test quantities is most pronounced in the case of a technically smooth surface condition of the profile, and it may be assumed that it is the result of the influence of the laminar flow regime which occurs in tests in the region of the leading edge.

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REFERENCE

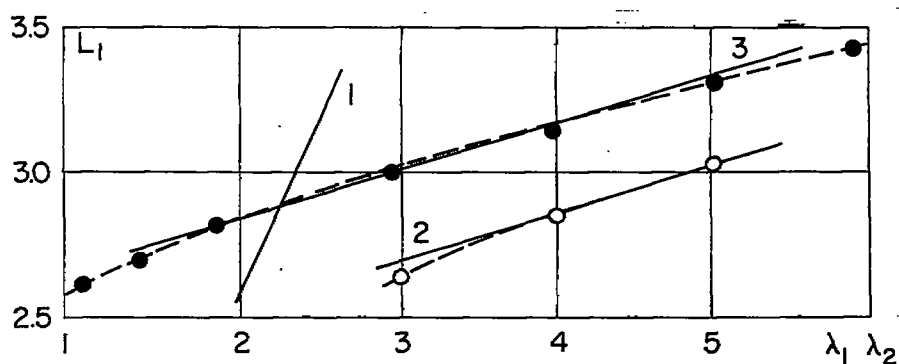1. Loĭtsianskiĭ, L. G.: Mekhanika zhidkosti i gazov [Mechanics of
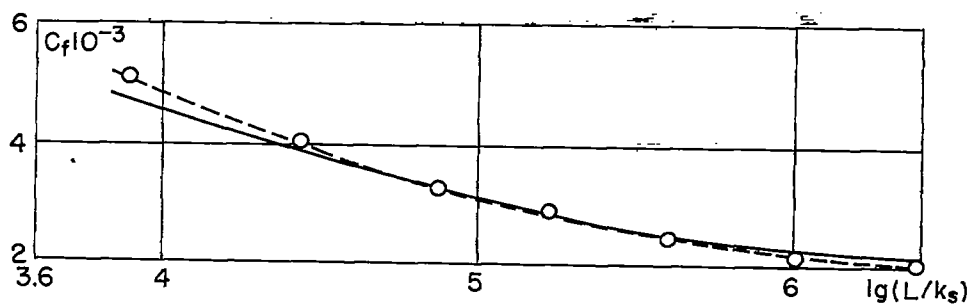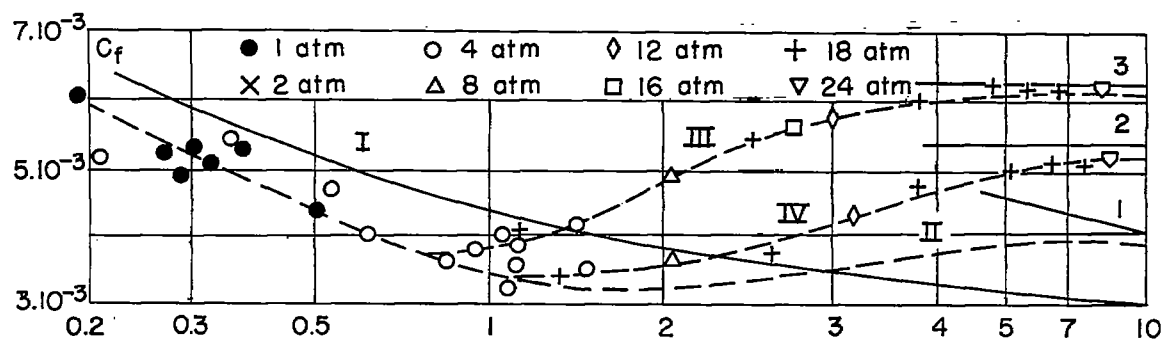   Liquids and Gases]. 1950.

Figure 1.

Figure 2.

Figure 3.